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of Mathematical Existential Statement Consequences on
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Community

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Abstract

This paper discusses the consequences of the latest PUEBI EYD V regulations for scientific ontological theorization through analyzing the semantical metaphysical commitment it reflects when we write formal mathematical statements using purely mathematical symbol (e.g., "there are 22 aardvarks"). This paper shows that PUEBI EYD V commits to mathematical Platonism metaphysically. This commitment brings harm to observable entities ontological nature in scientific theorization as shown in nominalism projects of philosophy of mathematics. Scientific theories - and even mathematical theories - should always reject the existence of independent objects for there exists only structures (as truth-value). Authors use nominalism stance as a methodology to reject the metaphysical commitment of PUEBI and defend the formal usage of writing mathematical statement without number symbols ("there are twenty-two aardvarks", etc.).

Keywords

PUEBI, Philosophy of Mathematics, Scientific Theorization, Mathematical Realism, Nominalism

1 Introduction; Logic operating in Formal Writing

Logic deals with the value-course of statements. In formal writing of PUEBI's EYD V, there lays hidden a value-course that is founded by particular logic when writing statements that asserts some "objects". Authors will assess this logical foundation. The updated PUEBI's EYD V sets out specific regulation on writing statements containing 'numbers' with two constituent words such as twenty-two to 22, ninety-one, 91, etc. Statements containing number or mathematical objects/entities assertion is called *mathematical existential statements*. We produce mathematical statements in both mathematic discourse and scientific discourse. Both receive different treatment regarding the ontological status of the mathematical objects, specifically *numbers*. For example, in mathematics, the basic arithmetic statements like " $2 + 2 = 4$ " asserts an existence of "4" as an abstract object existing independently when we write it with symbol.

This view is obtained by the Germany Logician Gottlob Frege (1848-1925). He distinguished the word "Number One" with "1" saying that one is indefinite and "1" is definite (Frege, 1950). Number symbols in math discourse (natural number, real number, integers, etc.) are committed with mathematical convention in mathematical theory. When we use that convention in mathematical theory, the epistemological aspect of the objects (number, sets, etc.) is dealing with no problem because we committed to it in such *definite* way (there's no essence in "14" anyways).

Now, the case is rather problematic when it comes to scientific discourse, can we view the word "1" or "2" in a definite way? First, we should focus on the intersection science has with mathematical convention regarding number as existing objects; in theorization, it usually uses mathematical convention to help them state/reveal nature of entities. In this objective, there's such a thing called "measurability" in unobservable statements. For example, we acknowledge properties of physics laws such as temperature and length that can be storable with *numerical information*. But here, without the number contained in the scientific properties, the entities continue to exist. What is asserted by statement "60 degrees Celsius" is not an

existing 60, but the heat as magnitude in the temperature law. In science, numbers should be regarded as properties of the existing-independently scientific entities, rather than the number itself that exist independently.

The definite ontological status of “1” in both mathematical and scientific discourse is operating upon a logical foundation. The conservative logic operating to conservative mathematic like arithmetic and sets (that advocates Platonism towards objects) is called the *second-order logic*, as advocated by Gottlob Frege. Objects in sets are to be represented by number. For example, when we assert an existing object, say, a person “Raisa” in sentence “There is Raisa”, then it is to be paraphrased with “there is 1 example of a human and that is Raisa”. The number always *asserts* that all concepts and objects exist in universal sets. The statement “There are 4 books” can be understood to mean “There are 4 objects that fall within the concept of books”. In this example, the detailed *nature/essence* of each book is to be denied for the objects (books) as they contain a logical foundation in a second-order way: from their concepts. The statement of a number is rather existential in this second-order sense, for “4” represents the real books. So, we can’t ask *how many* books there are for it implies “4” as objects of the books being observed rather than being the properties of quantities that the books have.

To assert something doesn’t exist is to assert the number 0. For example, there is 0 examples of a thing being considered *square-circle*, or both a square and a circle. This logical foundation is to be found in PUEBI regulation. Writing “given the temperature is 60 degrees Celsius” in scientific context is not to assert the object 60, but the properties of “heat”. Here number should not contain ontological import, but rather *epistemological* (how hot is the weather today?). To assert the nature of Mitochondria is to specify the detailed ontological status of its entities; as “the cell member that generate ATP as energy needed to power biochemical reactions”. Mitochondria is no substitutional with “1” and only detailed as member of the concept “organism cell’s member”, for example. This set-theoretical based logic of mathematical Platonism gives harm to scientific ontologizing (we will see how this logic implies in semantical problem in next section).

In *Intension and Decision*, R.M Martin writes ; “*The attention of the mathematician focuses primarily upon mathematical structure, and his intellectual delight arises (in part) from seeing that a given theory exhibits such and such a structure, from seeing how one structure is “modelled” in another, or in exhibiting some new structure and showing how it relates to previously studied ones... But the mathematician is satisfied so long as he has some “entities” or “objects” (or “sets” or “numbers” or “functions” or “spaces” or “points”) to work with, and he does not inquire into their inner character or ontological status. The philosophical logician, on the other hand, is more sensitive to matters of ontology and will be especially interested in the kind or kinds of entities there are actually....He will not be satisfied with being told merely that such and such entities exhibits such and such a mathematical structure. He will wish to inquire more deeply into what these entities are, how they relate to other entities...Also he will wish to ask whether the entity dealt with is sui generis or whether it is in some sense reducible to (or constructible in terms of) other, perhaps more fundamental entities*” (Martin, 1963)

Now, with the assessed metaphysical and logical commitment implied by PUEBI, we see how formal sentence writing (construction) using non-Platonism metaphysics and logic is used to defend the default ways of using “*ninety-nine*” in scientific statements. But of course, this is not to say writing “60 degrees Celsius” is prohibited, rather, this is to make possible our theorization of scientific entities which work the different way as in mathematics entities as R.M Martin said when writing *the observable statements* in theorization activity.

There are two different activities here: 1) Using number (mathematical entities) epistemic (for measurability/scale or constant in law) and 2) Using number as scientific entities properties as ontological use (for example, 60 is properties of heat, or an atomic number/property of oxygen is 8 is properties). In both activities, we reject commitment to mathematical entities. In the first activity, we can write number in 60 symbol (use mathematical convention) but use it for epistemic use (it has no logical issues), in second activity we should rather use linguistic convention of number (sixty-six etc) as possible for what are we produce in activity number 2 is that we derive ontologization of scientific entities (heat and oxygen) from theories that bound them (nominalist project) through observation statements. So when scientist making statements, theory is what give the entities its ontological nature, not mathematic. When writing statements about scientific entities, we would as possible reject the usage of number symbol. When writing today is 0 degrees doesn’t mean that there is no temperature operating, writing today is 0 degrees as observable statements is to be paraphrased as today temperature is zero degrees Celsius (if ,in this case, we want to use it to asserting existence of heat as entities, not for epistemic use).

In defending default ways of using “ninety-nine” instead of number symbol “99” as mathematical entities for scientific theorization of producing observation statements, author will use nominalist account as advocated methodologically by Hartry Field.

2 Linguistic vs Mathematical Convention on Ontologization of Objects

Before we get to how ontologization of scientific theorization supposed to work, let’s check how mathematical convention ontologization towards scientific entities supposed to work. In linguistic basis (in writing reality to language), we acknowledge two components of definitions containing object. The object, the first component, is called *definidum* and the second component is called *definiens*, it is used as the definition we give to the object (Copi, Cohen, & MacMahon, 2014).

In mathematical convention, every object that defined with mathematical convention supposed to be those and only those mathematical objects such as numbers, sets, function etc. In adopting Platonism, it is not only mathematical objects that is to be defined using mathematical convention, but also objects outside mathematics such as every object in universe in the *first-level sense*. This means that everything observable and unobservable is to be reduced with logic operating in mathematical convention that operates as well in defining number.

This is to be found in Frege’s work titled *Grundlagen Der Arithmetik* (Foundation of Arithmetic). Number n as object is to be justified with logic proof of “Equinumerosity” theorem. Objects in life (including observable and unobservable one) are also to be justified with this “Equinumerosity” theorem¹. Equinumerosity is the view that we can talk about object (singular term) q with different statements, such as a and b , iff both a and b are equinumerous or share the same direction (Frege, 1950). Now, if the meaning/semantic enterprise of objects works the same way as arithmetic meaning (“ $1 + 1 = 2$ ”), the justification or proof for the existence of “1” or “2” as n itself is that “each number can be defined in terms of its predecessor(s), since the natural number series up to a given number n has itself $n + 1$ members (since it starts from 0). This suggest the following general definition ($Nn+1$) where the number $n + 1$ is the number which belongs to the concept member of the natural number series ending with n ” (Frege, 1950).

The basic idea is that we have a and b to talk about. However if a can equal b , are we able to equate the earth’s axis to Caesar? Frege’s strategy to overcome the Caesar problem by defining numbers in terms of *value-courses* presupposes that the semantic rules of his system determine the reference of the value-course terms completely. In S10 of *Grundgesetze*, he raises the question whether the rule governing the two singular terms ‘ $\epsilon\phi(\epsilon)$ ’ and ‘ $\epsilon\psi(\epsilon)$ ’ have the same *bedeutung* iff the sentence ‘for all x : $\phi(x) = \psi(x)$ ’ is true and sufficient to determine the reference of each value-course term completely. Greimann answer is that it is not because, by appeal to it, “we can as yet neither decide whether an object is a value-course, if it is not given to us as such, or, if a value-course, of what function, nor in general decide whether a given value-course has a given property, if we do not know that this property is connected with a property of the function to which it belongs” (Greimann, 2003). Dirk Greimann said that this is still falls into the Caesar problem again for (A) does not fix the conditions for being the *bedeutung* of a given value course term completely because it does not fix the truth-values of mixed identity-statements such as $\epsilon - (\epsilon) =$ the truth value of the thought that for all x : $x=x$.

We see how number 1, 2, 3, etc. earned semantical accounts using logic proof, but are objects outside mathematics also to be justified with this logic proof? Of course, logic proof does no harm to mathematic² entities, but science has its own issues when adapting this number usage view with the logic given (because then logic has specific value-course construction), now this is where we should use linguistic convention instead. When creating *definiens* for *definidum* of arbiter objects in this world (either observable or unobservable), the linguist tends to adapt a nominalist account towards objects, in a way that every object (*definidum*) that is to be counted, is a nominal noun that can be given *definidum*. For example, when making a noun word for ‘table’ the object table is to be given a meaning not by some logic proof, but rather our ideas (mental construction) towards it. For example, table is to be defined as an idea of a thing that we use for “placing objects” or eating.

This nominalist view proceeds linearly, guided by the science behind it. Cognitive linguists define the process of nominalizing objects by the process of expressing our mind meaning through the objects. This

¹ See Frege on Sense and Reference

² Event though ,currently, mathematic logician finds second order logic irrelevant and use intuitionistic logic instead in advocacy of truth-value realism and not its entity.

is asserted in finding the object’s compositional path. That is, determining how an expression’s composite meaning relates to those of its components (Langacker, 2008)

In Platonism as we see in proof (1), the semantical account of objects, of any arbitrary object, is that it is logically provable by making it equinumerous with some extension of concept of table. This Platonist view towards objects give us classifications of philosophical stance towards object ontologization.

Table 1. Different views concerning mathematical objects conception

View on Objects (definidium)	Epistemic stance	Metaphysical Implication	Semantical account (definiens)
Platonism	Proof-based	Mathematical Realism/Entity Realism	None, reduced to value-course of theorem
Nominalism	Ideas (Mental Construction based)	Mathematical anti-realism, Anti-realism, Truth-value Realism	From ideas

As can be seen in table (1), defining an arbitrary object such as table requires no proof to instantiate the thing being-table to table through equinumerosity. The defining process is always up to human ideas generating what is it to be defined (in this theorization context, scientist). We see two opposing ways to perceive object; Platonic and nominalist. It is nominalist to perceive object is that view that there are no abstract entities that is non-mind-dependent to human. This means that our linguistic convention to nominalize things, objects, or other names existing in this world, is the basic means of its existence admission in a set-theoretic way.

Mathematic may use platonic or nominalist stance as it employs entities that are largely stable. However, it is of course a different case when it comes to scientific entities. The extension of nominalist stance on viewing objects will be discussed in the next section.

Here in figure (2) we see how using nominalist or Platonist, mathematical entities remain free of ontology deals. Even though, in some cases, adopting a Platonist stance may cause issues for a mathematician currently working on the truth-values justification of mathematical statements. So, they are focus on what makes mathematical statements either true or false and providing proof for such, rather than justification for what makes “16” exist.

Linguistics account on Number

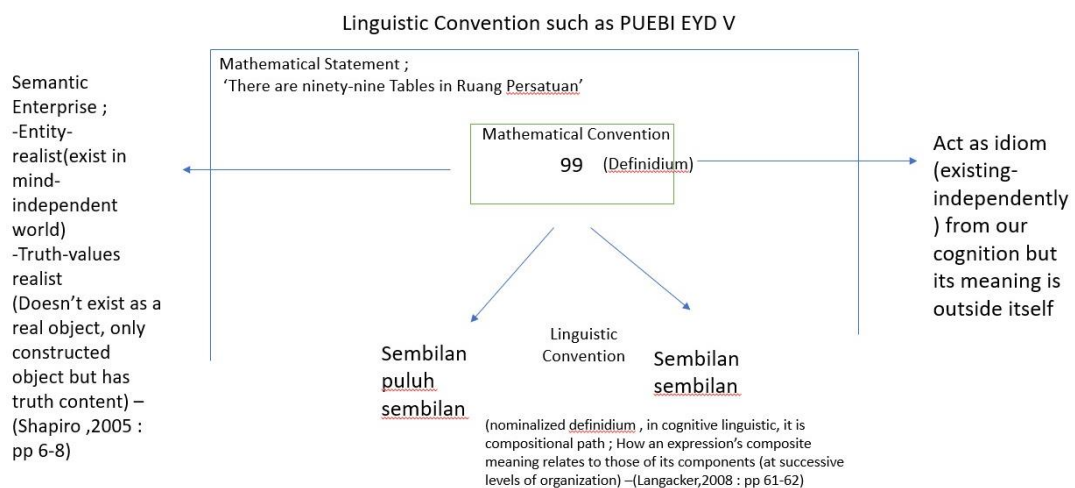


Fig. 2. The Nature of mathematical objects as definidium

Figure 2 shows the relationship between objects and linguistic vs mathematical convention to perceive the objects that will imply on its metaphysical stance. Entity realist denotes the platonic way, while truth-values realist stands for a nominalistic way to perceive objects.

In the next section we will see which views should be committed and how value-course in logical sense is to be reconstructed with new nominalist axioms to free scientific entities semantic enterprise from mathematic truth.

3 In Using Mathematical Fictionalism as Methodology

Fictionalism is similar to nominalism in the view that if entities work it does not cause entities to exist. Now that there are at least two ways on perceiving objects in linguistic sense (defining definidum), we will use the extension of nominalistic way that is its metaphysical stance in philosophy of mathematic as methodology to revise the prescriptive way on what to use for scientific ontologization in theorization. If Platonism in mathematic means treating numbers existing independently from our mind, then nominalism attempts to reject the existence of it as possible. In taking this metaphysical stance as methodology, author will use Hartry Field's work on Science without number.

First, Field argues that the utility of mathematical entities is structurally disanalogous to the utility of theoretical entities in physics. The utility of theoretical entities lies in two facts:

- (a) They play a role in powerful theories from which we can deduce a wide range of phenomena ; and
- (b) No alternative theories are known or seem at all likely which explain these phenomena without similar entities.

From these facts, it can be implied that scientific entities are indispensable from its theories. For example, subatomic particles, which is an entity indispensable from its theory. Field later argues that mathematic is quite the opposite, mathematical entities is dispensable from mathematic theories. Here he finds an account in which, somehow, mathematical existence-assertions are useful only in a limited context. This causes mathematic theory to be indispensable from its entities, ultimately making our physical phenomena inadequate. In fact, we want to regard it as literally true.

The usefulness of mathematical entities involves a feature of mathematic that is not shared by physical theories that postulate unobservables: "if you take any body of nominalistically stated assertions N , and supplement it with a mathematical theory S , you don't get any nominalistically stated conclusions that you wouldn't get from N alone. The analog for theories postulating subatomic particles is of course not true: if T is a theory that involves subatomic particles and is at all interesting, then there are going to be a lots of cases of bodies P of wholly macroscopic assertions which in conjunction with T yield macroscopic conclusions that they don't yield in absence of T ; if this were not so, theories about subatomic particles could never be tested" (Field, 2016).

Later, Field argues that conservative mathematical theories like sets is different from applied mathematics in a way that there is no way to apply them to the physical world. To use postulated abstract entities to the physical world, we need impure abstract entities, that is functions that map physical objects into pure abstract entities. Such impure abstract entities serve as a bridge between pure abstract entities and the physical objects; without the bridge, the pure objects would be idle. Consequently, if we regard functions as sets of certain sorts, then the mathematical theories we should be considering must include at least a minimal amount of *impure set theory*; set theory that allows for the possibility of urelements, where an urelement is a non-set which can be a member of sets. In fact, in order to be sufficiently powerful for most purposes, the impure set theory must differ from pure set theory not only in allowing for the possibility of urelements, it must also allow for non-mathematical vocabulary to appear in the comprehension axioms . So the 'bridge laws' must include laws that involve mathematical vocabulary and the physical vocabulary together.

Such bridge law is constructed by Field and it results in the conclusion that the bridge laws nature operating observably in physical theory is different than mathematical theory with observable; in the case of subatomic particles, the theory T can be applied to bodies of premises about observables in such a way as to yield genuinely new claims about observables, claims that would not be derivable without T . Whereas in the mathematical case the situation is very different. Here, if we take a mathematical theory that includes bridge laws, then that mathematics is applicable to the world. It is useful in enabling us to draw nominalistically statable conclusions from nominalistically statable premises. But here, unlike in the case of physics, the conclusions we arrive at by these means are not genuinely new. They are already derivable in a more long-winded fashion from the premises, without recourse to the mathematical entities (Field, 2016).

$N + S$ is a conservative extension of N , since N is nominalistic theory. It still may say things about the existence of abstract entities, so $N + S$ may well be inconsistent. But there a ways to to deal with this by either (1) introducing a 1-place predicate $M(x)$ in which 'x is a mathematical entity'; or (2) for any nominalistically stated assertion A , let A^* be the assertion that results by restricting each quantifier of A

with the formula 'not $M(x_i)$ ' and (3) for any nominalistically stated body of assertions N , let N^* consist of all assertions A^* for A in N . N^* is then an 'agnostic' version of N . For instance, if N says that all objects obey Newton's laws, but still allows for the possibility that there are mathematical objects that don't.

Whether similar points need to be made for our mathematical theory S depends on what we take S to be. If S is simply set theory allowing for urelements, no restriction on the variables is needed, since the theory already purports to be about non-sets as well as sets, we merely need to connect the notion of set that occurs with our predicate 'M' by adding the axiom $Ax(\text{Set}(x) \rightarrow M(x))$. If in addition the mathematical theory includes portions like number theory, considered as independent disciplines unreduced to set theory, then we must restrict all variables in them by a new predicate 'Number', and add the axioms $Ax(\text{Number}(x) \rightarrow M(x))$ and $Ex(\text{Number}(x))$. Presumably, however, everyone agrees that mathematical theories really ought to be written in this way (that is, presumably no one believes that all entities are mathematical), so I will not introduce a notation for the modified version of S . I will assume that S is written in this form from the start. Now it can be concluded the claim suitable for the relations of S and N :

Principle C ('for conservative'): Let A be any nominalistically statable assertion, and N any body of such assertions; and let S be any mathematical theory. Then A^* isn't a consequence of $N^* + S + 'Ex-M(X)'$ unless A is a consequence of N .

The derivation from the principle C is C' and C'' in which

Principle C': Let A be any nominalistically statable assertion, and N any body of such assertions. Then A^* isn't a consequence of $N^* + S$ unless it is a consequence of N^* alone.

This in turn is equivalent (assuming the underlying logic to be compact) to something still more obvious-sounding:

Principle C'': Let A be any nominalistically statable assertion. Then A^* isn't consequence of S unless it is logically true.

What can be understood from this is that many people see principle C'' as what many mathematical theories use. What is to be implied from this is that concrete entities is then logically incorrect. The same argument when using C' if mathematic with any body N^* (nominalistic assertion) implies assertion A^* which is logical consequence from N^* itself, so mathematical truth will rely to body of logically consistent assertion of $N^* + -A^*$ not being true. Maybe S in N that is consistent can be correct, but if it so, then failure in principle C will show that mathematic can't be true in all possible iterations. Standard mathematic could be not conservative, it could be inconsistent in a way that it's not conservative; that there are more than 1000 non-mathematic objects in universe. But that's just what field trying to conclude here; 'good mathematic is conservative, a discovery that accepts mathematic is not conservative is a bad discovery'.

The usage of mathematical existential assertion is limited in a specific context, because the nominalist consequence is to be reduced in its nominalist premises and this could be done not because they think the premise is true, but because mathematic knows that its truth-value lays on its nominalistic claim. The point is not to add existential assertion in science axiomatic system, but to use it wisely when we don't know how to axiomize science correctly. Mathematic still has to suffice the principle C. This principle is treated by Platonists as the essence of mathematic, that mathematic theory is true in all possible world, but in actuality this does not make a theory true.

It is now obvious how truth-values in mathematic is different than the truth of its abstract entities existence which is inconsistent. The principle C shows just how even nowadays mathematics (in applied mathematic), is no longer committing mathematic theory (S) that import finite abstract entities. Field continues to illustrate how mathematic theory is useful, but not true. In chapter two he argues for how only in context of arithmetic (supplying nominalistic body with arithmetic) with numbers making it easier for us to conclude the nominalistic body premises.

Let S : arithmetic of natural numbers.

Let N : theory that contains the identity symbol and the usual axioms of identity but does not contain any terms or quantifiers for abstract objects (there will not be singular terms like '87', instead, we use ' E_{87} ').

Now let's axiomatize:

$$\exists \geq 1 x A(x) \leftrightarrow \exists x A(x)$$

$$\exists \geq k x A(x) \leftrightarrow \exists x [A(x) \wedge \exists \geq j y (y \neq x \wedge A(y))],$$

where k is the decimal numeral that immediately succeeds j , and

$$\exists j x A(x) \leftrightarrow \exists \geq j x A(x) \wedge \neg \exists \geq k x A(x),$$

Pay attention that '87' is not of mathematical convention, it is rather used as part of an operator symbol. Now let's illustrate how Field applies (1) principle C (no inference come from no N) and from (2) N that facilitated with system S (the axiomatized logic that contain arithmetic) or $N + S$ and (3) from S alone.

From N alone ;

1. There are exactly twenty-one aardvarks (i.e., $\exists_{21}x A(x)$);
2. On each aardvark there are exactly three bugs;
3. Each bug is on exactly one aardvark, so
4. There are exactly sixty-three bugs.

From N with system S/ N + S (includes arithmetic of natural numbers plus set theory).

- 1' The cardinality of the set of aardvarks is 21
- 2'. All sets in the range of the function whose domain is the set of aardvarks, and which assigns to each entity in its domain the set of bugs on that entity have cardinality 3.
- 3'. The function mentioned in 2' is 1-1 and its range forms a partition of the set of all bugs, and
- 4'. The cardinality of the set of all bugs is 63

With S alone we can prove:

- (a) If all members of a partition of a set X have cardinality a, and the cardinality of the set of members of the partition is b, then the cardinality of X is a.b
- (b) The range and domain of a 1-1 function have the same cardinality, and
- (c) $3 \cdot 21 = 63$

Here 1-4 is provable in N + S. So, by principle C, 4 must follow from 1-3 in N alone. Now with or without writing formally as instructed in PUEBI the number symbol, our nominalist statements is to be the premise that gives our scientific theorization towards scientific entities new premises or conclusion. It is not the case that writing formally '67' will give harm, but for a better exploration towards N*, this S assertion should only works for supplement that help us simplify making N*. And S only operating in truth-values sense, means that mathematic is a priory true in its conservative theories. We view this theory true, but not necessarily believe that the entities is true. Besides, if we to use classic second-order logic for mathematic, in a way that committing mathematic entities should be true, it will cancel what Field says the many possibilities that there are 100000 more mathematical entities that is not discovered yet³.

4 Conclusions

The Figure 3 shows how (1) theorization both in science and mathematic supposed to be done in a nominalist stance vs (2) how theorization is to be done in a platonistic stance as advocated by PUEBI'S EYD V.

Problem in Frege and PUEBI to Semantic Convention

If we use *mathematical convention* in *linguistic convention*, then it will prevent our 'defining' or 'theorization' activity towards observability of science entities. PUEBI Convention on writing Sembilan puluh Sembilan as 99 subsume *our semantic enterprise* on theorization to *mathematic semantic enterprise*.

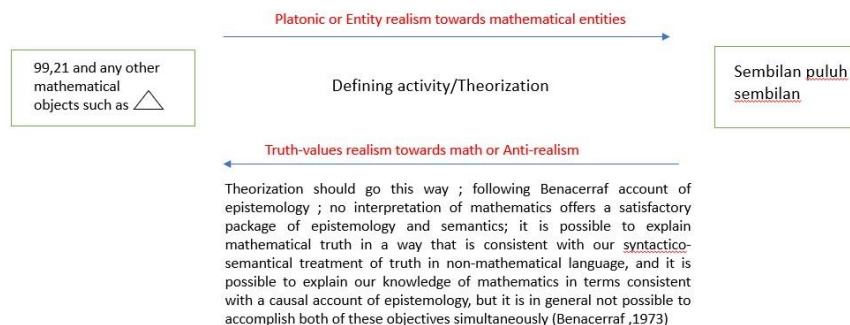


Fig. 3. Strategy on Theorization

³ Nowadays this entities and truth is to be constructed using modern logic

Theorization in linguistic basis should be performed naturally as shown in nominalist behavior. Any linguistic convention that tries to cancel our natural way on ontologizing (nominalize) objects as definidium is an obstacle to scientific activity. This is the problem of philosophy of language: the way we talk about world through language activity has its own rigid epistemological and metaphysical rule and committing to one rule will affect any product of the scientific process. This is justified in section 2, where numbers are used to represent objects that provide nothing but syntatico-semantical treatment of truth. It can be concluded that this paper found that PUEBI's EYD V (1) has logic and metaphysical import on committing to mathematical Platonism. (2) this regulation will give problems in doing theorization for the (1) imports will give obstacles including epistemological and ontological treatment of semantic truth in defining scientific entities (definidium).

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